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Finite-size scaling of the superfluid density in two-dimensional superfluids *

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Using the cluster Monte Carlo algorithm we study the helicity modulus in the $x-y$ model on two-dimensional $L \times L$ lattices. We calculate the renormalization group beta function and our results for the helicity modulus obey conventional finite-size scaling theory and are in agreement with the Kosterlitz-Thouless theory.

The superfluid transition in two-dimensional (2D) liquid helium is understood as a Kosterlitz-Thouless[1] phase transition. The temperature dependence of the superfluid density has been measured[2] and its main features agree with theoretical predictions[3]. There are, however, experimental results for the superfluid density[4] in helium films of finite thickness h indicating that a straightforward application of finite-size scaling theory[5] with respect to h cannot be applied. Further insight is expected to be gained from experiments on confined helium and in microgravity conditions planned for the near future[6].

In this paper we study finite-size scaling of the superfluid density, in finite 2D helium films of size $L \times L$, with respect to L. In order to calculate the superfluid density in such finite-size films, we shall use a recently developed[7] Monte Carlo updating technique (the so-called "cluster Monte Carlo") which effectively deals with the "critical slowing down". We have determined the renormalization group beta-function and our scaling transformation collapses the calculated superfluid density for various lattices on one universal curve. Our results obey finite-size scaling using values for the critical exponents close to those calculated by Kosterlitz and Thouless.

We consider the Hamiltonian for the two-dimensional planar $x-y$ model $H[\vec{s}(\vec{r})] = J\sum_{<i,j>} \vec{s}_i \cdot \vec{s}_j$ where $\vec{s} = (\cos\theta, \sin\theta)$. In this paper we present results for the helicity modulus $Y(T, L)$ (which is proportional to the superfluid density) calculated on $L \times L$ size lattices with periodic boundary conditions. Our computer program has been checked by reproducing within statistical errors results of other simulations for various quantities for the 2D and 3D $O(N)$ invariant models. In Fig.1, we present our results for the helicity modulus as a function of the temperature T/J as obtained for $L = 20, 30, 40, 60, 80, 100$. The helicity modulus

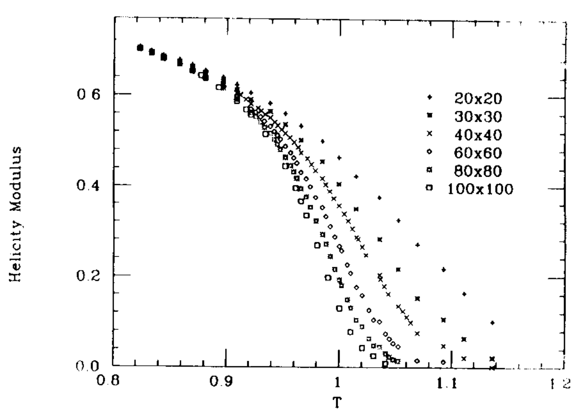

Figure 1. The helicity modulus as a function of T for various lattices $L \times L$

$Y(L, T)/J$ for a 2D $x-y$ model is dimensionless and, thus, it should be kept constant under scaling transformations. Namely, the beta function can be obtained by defining a function $T = F(L)$ such that $Y(L', F(L')) = Y(L, F(L))$. We define the beta function as:

$$\beta(T) = -L\frac{dT}{dL}, \qquad (1)$$

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In Fig. 2 we present our results for the beta function as obtained via this transformation taking into account all calculated values for $Y(L,T)$. Following the implications of the Kosterlitz-Thouless theory, we have fitted the curve of Fig. 2 with a function of the form

$$\begin{aligned} \beta(T > T_c) &= c(T - T_c)^{1+\nu} \\ \beta(T \leq T_c) &= 0. \end{aligned} \quad (2)$$

The result of the χ^2-fit is shown by the solid line. We obtained $T_c = (0.86 \pm 0.014)J$, $\nu = 0.58 \pm 0.21$ and $c = 0.95 \pm 0.24$.

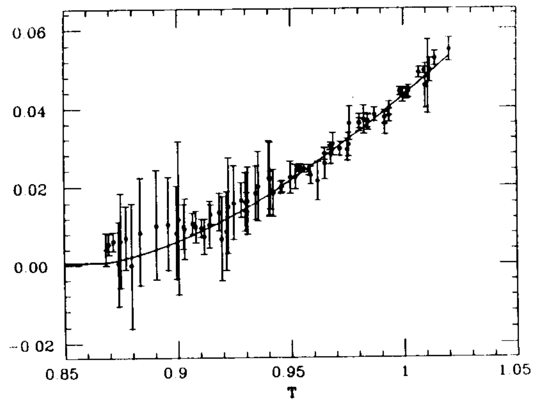

Figure 2. The beta function as a function of T. The solid line is the result of the fit to the form (2)

Equating the right-hand-sides of Eqs. (1) and (2) and solving the resulting differential equation for the function $T = F(L)$ we obtain:

$$\ln(L) - \frac{b}{(T - T_c)^{\nu}} = a \quad (3)$$

where $b = 1/(\nu c)$ and a is an integration constant. This equation defines the scale transformation under which the observables remain invariant. In Fig.3 we plot our calculated values of $Y(T,L)$ versus $a = \ln L - b(T - T_c)^{-\nu}$ for various size lattices. Apart from a constant, a is also the logarithm of the ratio of $L/\xi(T)$ where $\xi(T)$ is the Kosterlitz-Thouless correlation length. Notice that to a good approximation all curves for various size-lattices collapse on the same universal curve. Therefore finite-size scaling is satisfied for the helicity modulus of the 2D $x - y$ model.

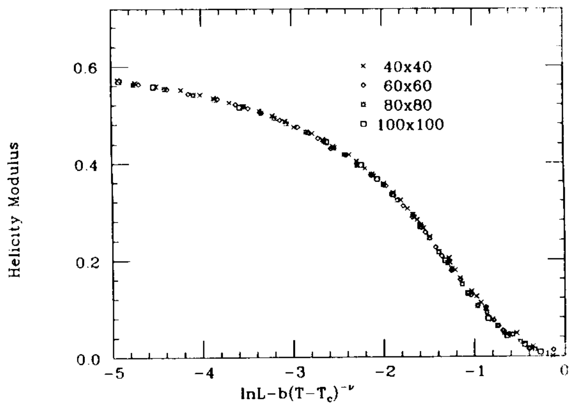

Figure 3. Using the scaling transformation (3) our results for the Helicity modulus for various size-lattices collapse on the same curve

The helicity modulus, in the $L \to \infty$ limit will be a discontinuous function at $T = T_c$. Notice, however, that, this jump is rounded dramatically in finite-size systems and according to (3) the temperature width δT of this rounding decreases with increasing L very slowly, namely $\delta T \sim 1/(\ln L)^{1/\nu}$.

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