

A Quantum-Dot Array as Model for Copper-Oxide Superconductors: A Dedicated Quantum Simulator for the Many-Fermion Problem

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Quantum systems with a large number of fermionic degrees of freedom are intractable by quantum simulations. In this paper we introduce the concept of a dedicated quantum simulator (DQS) which is an artificial system of quantum dots whose Hamiltonian maps exactly to the original many fermion problem. While the universal quantum simulator (UQS) introduced by Feynman in 1982 can simulate any quantum mechanical many-body problem, a DQS can only solve a particular many body problem. Our concept of the dedicated quantum simulator is not a quantum computer but rather a quantum “analog” device, dedicated to a particular quantum computation. As an example, we consider the system of the CuO plane in the copper-oxide superconductors and we propose an array of electrostatically confined quantum dots to be used as its dedicated quantum simulator. We show that this dedicated device can be used to image stripe formation as a function of the electron doping using electric force microscopy. We argue that such a dedicated quantum simulator may be easier to realize in the future compared to a general purpose quantum computer.

1. INTRODUCTION

In simulations of quantum many-fermion systems large statistical fluctuations arise due to cancellations among large amplitudes of configurations differing by fermion exchanges. As a consequence, the computational time required to obtain acceptable statistical errors grows exponentially with the system size. This limitation is severe because only small-size systems can be simulated and that prevents us from being able to extrapolate to the thermodynamic limit. This is a general problem in several fields of computational physics and chemistry and it would be of general importance if a

computational instrument that simulates quantum fermion systems could be constructed.

Progress in quantum computation¹ has raised hopes that a realistic computation using a quantum computer could be achieved in the future. Feynman had conjectured³ that the quantum computer can be used to simulate any local quantum system. Later, Lloyd showed⁴ that a quantum computer can be programmed so that it can be such a universal quantum simulator (UQS). Recently DiVincenzo *et al.*² have proposed that a coupled quantum-dot pair may be used to represent a q-bit. However, in any attempt to design a general purpose quantum computer one needs to find an approach to externally manipulate quantum mechanical states, to preserve the quantum coherence of these states for some time and to transport them at a macroscopic distance away before the quantum information is dissipated.

In this paper we introduce the concept of a dedicated quantum simulator (DQS) in contrast to the UQS. It will become clear that a dedicated quantum computer to simulate a specific many-fermion model does not require a controlled initial state or building quantum gates which also need to be controlled. A DQS as defined in this paper, while it should be a system without significant amount of impurities or other defects which could create decoherence, it does not require from us to manipulate q-bits where destruction of coherence can also occur. Thus, building a DQS is a more realistic goal in comparison to building a universal quantum computer.

Our concept of the dedicated quantum simulator is not a quantum computer but rather a quantum “analog” device, dedicated to a particular quantum computation. Long time ago, before digital classical computers became fast enough to carry out numerical integration or differentiation, the so-called analog computers were used for that purpose. To obtain the integral or the derivative of a function $f(t)$, an electrical time-dependent current, which changes in time in the same fashion as the function $f(t)$, was used as input to a circuit which contains a capacitor or an impedance. For the integral or the derivative of the input function one would measure the voltage across the capacitor or across the impedance respectively. No digital computation was carried out by such a device, but the entire “computation” was based on the physical property of the device used. A particular physical circuit was dedicated to a specific computation, i.e., the capacitor circuit for integration, while the impedance circuit for differentiation. In this paper we introduce a dedicated quantum simulator whose relationship to a quantum computer is analogous to the relationship of a digital classical computer to an analog classical computer. Notice that we are careful and we use the term “simulator” as opposed to the term “computer.”

It is well known in statistical mechanics that the Landau–Ginzburg model of a superfluid can be mapped onto the X – Y model. The X – Y model can also describe the critical fluctuations of certain types of magnetic systems where the order parameter is a two component vector. Thus, instead of using a digital computer to compute the critical exponents associated with the superfluid to normal-fluid phase transition, one can study the experimental results obtained on such a designed magnetic system assuming that the connection (the mapping) of the magnetic system to the X – Y model is accurate. One could therefore think of this model magnetic system as a *dedicated simulator* of the critical properties of the superfluid.

The idea can be extended further to a pure quantum many body system where we are interested in the statistical properties of that system. If we could prepare a physical system which is described by a known model quantum Hamiltonian, all we would have to do is perform measurements of the desired observables. It is rather hopeless to expect that we could configure atoms together to interact in our desired way as in the model Hamiltonian. Quantum dots share many features with the atomic spectra and they are sometimes called “artificial” atoms. The parameters defining a quantum dot can be artificially controlled and designed. In addition, one can create arrays of such dots where we can manipulate their interactions. Therefore, if we could design an array of quantum dots interacting in a similar way, the original physical system can exist at a very different energy scale but as long as the model system (used in the simulation) shares the same geometry and the same values of doping and dimensionless parameter ratios one can directly compare dimensionless ratios of observables using scaling. This is what we call “dedicated quantum simulator” and has nothing to do with the functions involved in quantum computing, just in the same way as the question of how a classical digital computer works is irrelevant to the problem of an analog computation.

Making a quantum computer that performs operations which are controllable at the so-called q-bit level is a far more difficult task than making a device that can perform such a dedicated task to solve a specific quantum many fermion Hamiltonian. The reason is that every nature’s operation is quantum mechanical and thus we can “take advantage” of that and instead of breaking down the problem into a huge set of classical operations we prepare a many-body quantum system which is described by, and thus can mimic exactly, the theoretical many-body Hamiltonian which we wish to solve.

In this paper, we choose to give an example of a two-dimensional quantum-dot array which can be mapped to a Hubbard-like Hamiltonian identical to that used to describe the physics of the Cu–O plane of the high temperature superconductors. This serves as model for the copper-oxide

plane in the copper-oxide superconductors based on a quantum dot two-dimensional array. The Hamiltonian which describes the quantum-dot array does not contain phonons as degrees of freedom and thus, one can determine the physical properties in the absence of phonons. The quantum-dot system exists at an energy scale (a few meV) which is three orders of magnitude smaller than that of real physical system. We think of this system of the array of quantum dots as a quantum simulator of the physics of the original system. We also discuss that this system should form stripes at the appropriate filling factor in an analogous manner to that in the copper-oxide superconductors. In addition, we discuss how to use this model to study the formation of stripes in the original problem of the copper-oxide planes.

2. THE QUANTUM DOT ARRAY

We wish to consider a two-dimensional electron gas (2DEG) which forms in an $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$ heterostructure. Such a heterostructure can be grown by molecular beam epitaxy (MBE) on a n^+ doped GaAs substrate. On top of this layer one grows a layer of pure AlGaAs. Next, a layer of pure GaAs is grown which has smaller gap than AlGaAs. The 2DEG is formed in this last layer near the interface with the AlGaAs layer. A positive voltage applied to the n^+ doped substrate controls the density of the 2DEG. Two-dimensional electron densities of the order of $n = 10^{11}/\text{cm}^{-2}$ are desirable for the application described in this paper. A spacer of pure GaAs between the substrate and the AlGaAs material may also be necessary to increase the mobility of the electrons at the interface.

In this paper, we consider the case of the CuO plane of the copper-oxide superconductors. For this example, we propose the metallic gate with the hole pattern shown in Fig. 1, i.e., with an array of two different size holes placed onto the heterostructure as the top electrode. Such patterns can be "drilled" on a thin metallic plate with e-beam lithography. A negative gate voltage V_g is applied between this gate and the 2DEG.

First, let us consider a single hole of radius a created on the metallic gate. At a distance d from the gate, the potential is modified from its value in the absence of the hole by an amount $\delta V_a(\vec{r})$ given by Ref. 7

$$\delta V_a(\vec{r}) = -\frac{Ea^2}{\pi} \int_0^\infty dk j_1(ka) e^{-kd} J_0(kr), \quad (1)$$

where $E = V_g/(\epsilon d)$ is the electric field below the top metallic gate in the absence of the hole and ϵ is the dielectric constant for pure GaAs. Here we

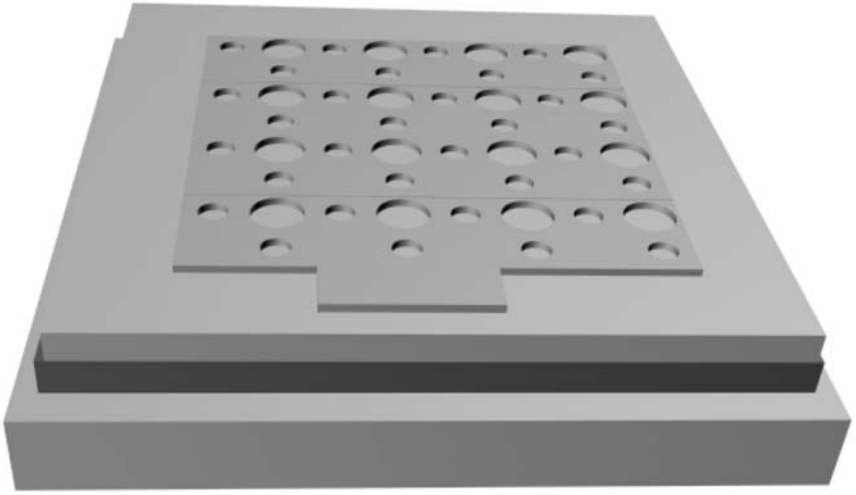


Fig. 1. Quantum dot model for the Cu-O plane of the copper-oxide superconductors.

have considered a cylindrical coordinate system with the z axis perpendicular to the plane and passing through the center of the hole with $r = \sqrt{x^2 + y^2}$ the distance from the axis. The electrons are thus trapped in a quantum well and as they move away from $r = 0$ they feel a parabolic repulsion which for small r is given by

$$\delta V(\vec{r}, a) = \delta V(0, a) + \frac{1}{2} m^* \omega_0^2(a) r^2 \quad (2)$$

$$\omega_0(a) = \sqrt{\frac{|e| E a}{\pi m^*}} \frac{a}{(d^2 + a^2)} \quad (3)$$

$$\delta V(0, a) = \frac{-|e| E a}{\pi} \left(1 - \frac{d}{a} \tan^{-1} \left(\frac{a}{d} \right) \right), \quad (4)$$

where m^* is the electron effective mass. We consider a metallic gate with the array of holes shown in Fig. 1 which produce a modification of the external field at the interface which is the superposition of the change caused by each hole:

$$V(\vec{r}) = \sum_{\vec{R}, \vec{\sigma}} (\delta V(\vec{r} - \vec{R}, a_1) + \delta V(\vec{r} - \vec{R} + \vec{\sigma}, a_2)) \quad (5)$$

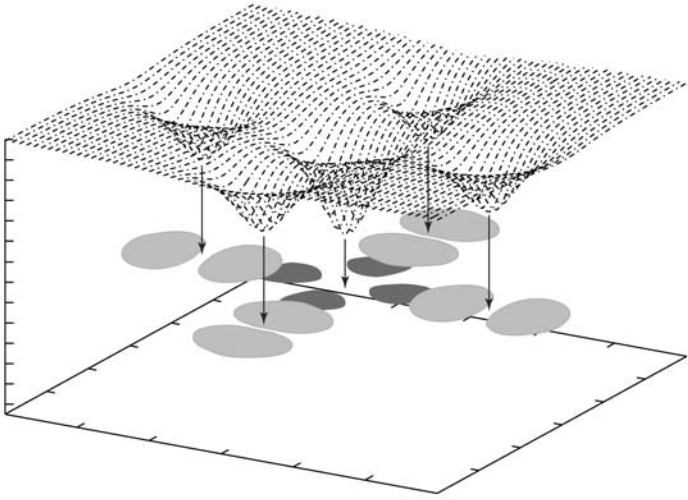


Fig. 2. The potential landscape for a cluster of quantum dots with the central dot somewhat larger than the other four surrounding dots. Below the potential minima the orbitals of the outer electrons, i.e., the $d_{x^2-y^2}$ of the central dot and the p_x and p_y of the surrounding smaller dots are shown. Inside the shaded areas the magnitude of the wavefunction is larger than half its peak value.

where $\vec{R} = (n_x \hat{x} + n_y \hat{y}) b$ spans the entire square lattice of lattice spacing b formed by the centers of the larger holes of radius a_1 . The smaller holes of radius a_2 are at positions $\vec{R} + \vec{\sigma}$ where $\vec{\sigma}$ takes the values $b/2\hat{x}$ and $b/2\hat{y}$. This potential can be considered as an external field felt by the 2DEG and to which the electrons will respond. If the negative gate potential is not strong enough to cause total depletion of the heterojunction from the 2DEG, the electronic charge of the heterojunction will move to benefit from the less repulsive potential near the holes. The self-consistent potential landscape on the heterojunction will depend on the 2D electron density and is expected to look schematically as the one shown in Fig. 2 for a cluster of one hole surrounded by 4 smaller ones.

We will assume that the electrons in the dots feel a harmonic oscillator potential. Clear experimental indication⁸ that electrostatically confined quantum dots feel parabolic confinement comes from the magic numbers observed. Let us denote by $|n_x, n_y\rangle$ the eigenstates of the 2D harmonic oscillator in Cartesian coordinates. In the case where we have square lattice symmetry the circular symmetry of the “atomic” potential is reduced and we need to consider irreducible representations of the group C_{4v} . In our illustrative example of the array the dots have at most 12 electrons per dot. In these cases we will need only the following orbitals:

(i) The state with lowest energy is the state with s -wave symmetry given by

$$\langle \vec{r} | s \rangle = \langle \vec{r} | n_x = 0, n_y = 0 \rangle = \sqrt{\frac{\lambda}{\pi}} e^{-\lambda r^2/2}, \quad (6)$$

where $\lambda = m^* \omega / \hbar$.

(ii) The next excited states are the two degenerate p states. The p_x state given by

$$\langle \vec{r} | p_x \rangle = \langle \vec{r} | 1, 0 \rangle = \sqrt{\frac{2}{\pi}} \lambda x e^{-\lambda r^2/2}. \quad (7)$$

The state $|p_y\rangle = |0, 1\rangle$ obtained by replacing x with y .

(iii) The d wave states are also separated according to the representations of the C_{4v} .

(a) The state $|d_{x^2+y^2}\rangle = \frac{1}{\sqrt{2}} (|2, 0\rangle + |0, 2\rangle)$ belongs to the representation Δ_1 and it is given by

$$\langle \vec{r} | d_{x^2+y^2} \rangle = \sqrt{\frac{\lambda}{\pi}} (1 - \lambda r^2) e^{-\lambda r^2/2}. \quad (8)$$

(b) The state $|d_{x^2-y^2}\rangle = \frac{1}{\sqrt{2}} (|2, 0\rangle - |0, 2\rangle)$ belongs to the Δ_2 representation and it is given as

$$\langle \vec{r} | d_{x^2-y^2} \rangle = \frac{\lambda^{3/2}}{\sqrt{\pi}} (y^2 - x^2) e^{-\lambda r^2/2}. \quad (9)$$

(c) The state $|d_{xy}\rangle = |1, 1\rangle$ belongs to the Δ_y representation and it is given by

$$\langle \vec{r} | d_{xy} \rangle = 2\lambda \sqrt{\frac{\lambda}{\pi}} x y e^{-\lambda r^2/2}. \quad (10)$$

Now we wish to consider the square lattice arrangement of quantum dots presented in Fig. 1 with quantum dots of two different radii a_1 and a_2 with $a_1 > a_2$. In the smaller dot the potential starts from a higher value at its center. The spacing between the energy spectra depends on the frequencies $\omega_0(a_1)$ and $\omega_0(a_2)$. By controlling the relative depth of the potential in the dots (by changing V_g and a_1 and a_2) and the density of the 2DEG

(using the back-gate voltage), we can create the following situation. The larger dot is filled with 12 electrons in the $s^2p^4d^6$ pseudo-atomic configuration and the smaller dot with 6 electrons in the s^2p^4 pseudo-atomic configuration. Thus the highest occupied levels are the d for the large dots and the p for the small dots. Next, we show how this can be achieved.

We need to estimate the required quantum-dot sizes and 2D electron densities necessary for producing the case discussed in the previous paragraph. We need to satisfy the following condition:

$$\delta V(0, a_1) + \mu(N_1, a_1) = \delta V(0, a_2) + \mu(N_2, a_2) \quad (11)$$

where $\mu(N, a) = E(N, a) - E(N-1, a)$ is the chemical potential for each of the quantum dots in the presence of only the quadratic term of the interaction in Eq. (2). Here $E(N, a)$ is the total dot energy as a function of the electron number and we need to distinguish $E(N, a_1)$ from $E(N, a_2)$ because of the two different dot sizes.

3. A MODEL FOR A SINGLE QUANTUM DOT

There are several calculations for a single quantum dot using various approximations.⁹⁻¹² These calculations have been carried out using a fixed value of the external parameter ω_0 of the harmonic confining potential. Our problem here is more complex because for a given value of a_1 we need to determine a_2 required to satisfy Eq. (11) and this requires the knowledge of the full function $E(N, a)$. Next, we present a simple model to express the energy of one dot in a harmonic potential of external frequency $\omega_0(a)$. We represent the N -dot wave-function as a Slater-determinant of Hermite polynomials which correspond to a 2D harmonic oscillator potential of "dressed" frequency ω . The value of ω will be determined by minimizing the expectation value of the Hamiltonian which includes the Coulomb interaction. The presence of the Coulomb interaction will decrease the value of ω compared to ω_0 . We find that

$$E(N, a) = \frac{1}{2} \hbar \left(\omega + \frac{\omega_0^2}{\omega} \right) n(N) + u \sqrt{\hbar \omega} \frac{N(N-1)}{2}. \quad (12)$$

The values of the function $n(N)$ for N ranging from 0 through 12 are 0, 1, 2, 4, 6, 8, 10, 13, 16, 19, 22, 25, 28 respectively. The last term corresponds to the electron-electron interaction which scales as $N(N-1)/2$ with respect to N . In addition, it is inversely proportional to the dot size which implies that it is proportional to $\sqrt{\omega}$. The parameter u gives a measure of the Coulomb interaction in the dot when all the important dependences are scaled out and we expect it to be almost independent of ω and N . In the

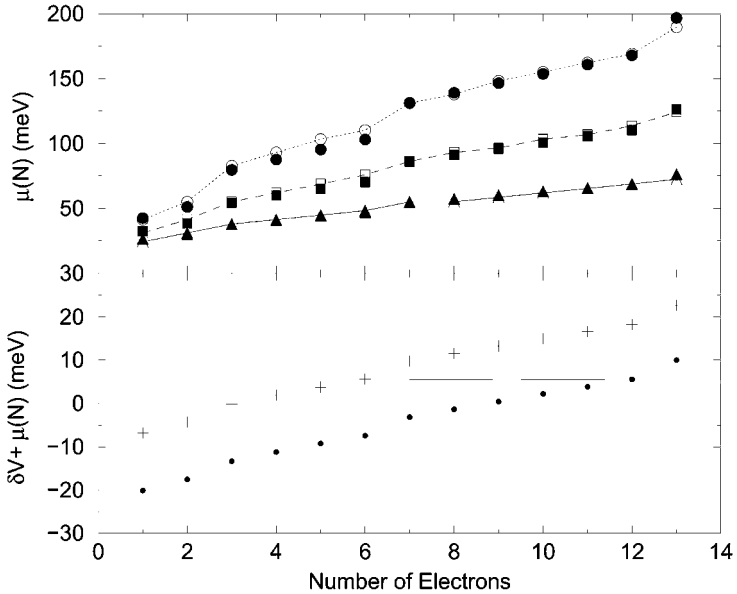


Fig. 3. Top: The results of our model calculation for quantum dots (solid symbols) are compared with results of DFT calculation (open symbols) for the cases of $\hbar\omega_0 = 4$ meV (triangles), 10 meV (squares) and 20 meV (circles). Bottom: The chemical potential of the two different size dots match for electron numbers 6 (+ signs) and 12 (solid circles).

capacitance model for large dots, u is a constant. Assuming a universal value for u for any dot in a parabolic potential, for a given value of ω_0 and N the energy is minimized with respect to ω . In the top part of Fig. 3, the results of our calculation of $\mu(N)$ are compared with those of a recent density function theory (DFT) calculation¹² using $u = 2 \text{ meV}^{-1/2}$ and the optimal value of ω . The agreement is very satisfactory given the fact that the same value of u is used for the results obtained for three very different values of $\omega_0 = 4, 10, 20$ meV.

4. DETERMINATION OF THE PARAMETERS OF THE QUANTUM-DOT ARRAY

We take $\epsilon = 12.9$ and $m^*/m = 0.067$ for *GaAs* and we choose $N_1 = 6$, $N_2 = 12$, $d = 500 \text{ \AA}$, $a_1 = 1000$ and $V_g = 1 \text{ V}$ and we find that, $\hbar\omega_1 = 1.915 \text{ meV}$ and in order to satisfy Eq. (11) we need to take $a_2 = 620 \text{ \AA}$, and $\hbar\omega_2 = 1.843 \text{ meV}$. The left and the right hand side of Eq. (11) are shown in the bottom of Fig. 3. We note that the horizontal solid line denotes that the two functions share the same value for $N_1 = 6$ and $N_2 = 12$. Notice that the

chemical potential differences $\delta\mu(N) = \mu(N) - \mu(N)$ are: 1.69, 1.64, 4.46 meV for the larger dot and $N = 11, 12, 13$ and 1.93, 1.81, 4.13 meV for the smaller dot and $N = 5, 6, 7$.

We can tune the doping (controlled by the back-gate potential) to fill the two dots with $N_1 = 12$ and $N_2 = 6$ electrons. This can be achieved by controlling the total 2D electron density to be $24/b^2$ and taking $b = 3400 \text{ \AA}$ (b should be larger than $2(a_1 + a_2) \simeq 3240 \text{ \AA}$ for our example here), this corresponds to a 2D density of $n = 2.0 \cdot 10^{10}/\text{cm}^2$. If we reduce the value of the electron density further we can change the filling of the p -level of the small dots and of the d -level of the larger hole. By changing V_g , and a_1 , a_2 and d we can increase the ω_0 's which will increase the required 2D electron density.

In order to describe the electron hopping from a p -level of the small dot to the $d_{x^2-y^2}$ level of the neighboring larger dot we will introduce the hopping matrix element $V_{pd} = \langle p_x | i H | d_{x^2-y^2} \rangle_{i+\hat{x}} = \langle p_y | i H | d_{x^2-y^2} \rangle_{i+\hat{y}}$. Notice that the hopping matrix elements between p and d_{xy} of neighboring dots is zero, while that between p and d_{1-r^2} is smaller and can be neglected for appropriately chosen inter-dot distances. These outer electron orbitals with significant overlap integral are shown in Fig. 2. Notice the direct correspondence of the orbitals here and the Cu $d_{x^2-y^2}$ and the oxygen p_x and p_y in the Cu-O plane of the copper-oxide materials. Furthermore, the orbital $d_{3z^2-r^2}$ of Cu in the Cu-O plane, whose role was much debated, corresponds to the orbital d_{1-r^2} .

The tight binding Hamiltonian describing this quantum dot array is

$$H = \sum_{i,l \in (i), \sigma} (\epsilon_d d_{i\sigma}^\dagger d_{i\sigma} + \epsilon_p p_{l\sigma}^\dagger p_{l\sigma} + V_{pd} d_{i\sigma}^\dagger p_{l\sigma} + H.c.) \quad (13)$$

and can be analytically diagonalized in a straightforward manner. Here $\sum_{l \in (i)}$ denotes the sum over the neighbors of site i . $d_{i\sigma}^\dagger$ and $p_{l\sigma}^\dagger$ create electrons in the states $|d_{x^2-y^2}\rangle_i$ and $|p_x\rangle_l$ (or $|p_y\rangle_l$) respectively with spin σ . Large overlap can be achieved when the inter-cell distance b is comparable to $b_o \sim 4(\lambda_1^{-1/2} + \lambda_2^{-1/2})$. In the case of the example we gave above $\lambda_1 \simeq \lambda_2$ and $b_o \sim 2000 \text{ \AA}$. Taking $\lambda_1 = \lambda_2 = \lambda$, the overlap integral between the p and the d states which are separated by a distance b decays as $e^{-(b/b_o)^2}$. Thus, the magnitude of V_{pd} can become a large fraction (of the order of 20%) of the within-dot electron kinetic energy $\hbar^2\lambda/2m^* \sim 1 \text{ meV}$.

Based on experimental results on charging of a quantum dot using a capacitor⁸ and our results here one needs to include in the Hamiltonian a term of the form

$$H_U = U_d \sum_i d_{i\uparrow}^\dagger d_{i\uparrow} d_{i\downarrow}^\dagger d_{i\downarrow} + U_p \sum_{i,l \in (i)} p_{l\uparrow}^\dagger p_{l\uparrow} p_{l\downarrow}^\dagger p_{l\downarrow}. \quad (14)$$

$U_{d,p}$ are of the order $\delta\mu_1(12)$ or $\delta\mu_2(6)$ both of which are ~ 2 meV. Including this term in the above tight-binding Hamiltonian we obtain the same two-band Hubbard model which has been used to describe a single layer of copper-oxide. Short as well as long range Coulomb interaction should be included in the above Hamiltonian in order to understand the phase diagram of such an array of quantum dots. However, even when one attempts to describe the physics of the Cu-O planes in the copper-oxide superconductors one should include these interactions.

5. STRIPE FORMATION ON THE ARRAY AND THEIR IMAGING

The copper-oxide materials exhibit¹³ stripe formation at filling factor of $1/8$. Numerical studies^{14–16} of the t-J model, which is a possible reduction of the above three-orbital model,¹⁷ indicate that the model has a phase separation instability. Some different numerical studies¹⁸ of this model indicate that the system at least with cylindrical boundary conditions seems to form stripes at the appropriate filling factor and value of J/t . Independently of the controversy surrounding the stripe formation in the t-J model,^{14,19} it may be expected that stripes form when the long range part of the Coulomb interaction is included in the t-J model. At the appropriate filling factor one might expect formation of stripes in the quantum dot system described previously. The quantum dot array discussed here cannot allow a macroscopic electronic charge separation, thus, we expect (just like in the real copper-oxide superconductors) to see a striped state or another form of clustering of charge and spin.

The formation of the stripe state can be investigated using the quantum dot array proposed in the present paper. We expect that the stripe state can be detected by electric force microscopy (EFM). Since the inter-dot distance is of the order of $0.1 \mu\text{m}$, atomic scale spatial resolution is not required. However, we need to be able to detect electric potential variation on the surface of the quantum dot array of the order of 1 mV or smaller. Therefore a special purpose tip coated with a metal layer should be made which should be wider than the typical size in order to detect the voltage change associated with such inhomogeneous charge distribution when stripe formation occurs.

Transport measurements can be also performed which can possibly shed light on the original problem. Conductance measurements have already been performed on one-dimensional arrays of quantum dots.²³ There are indications²⁴ that an Anderson–Mott metal-insulator transition might have been observed.^{23,25} At filling factors around one hole per unit cell, one expects to find an antiferromagnetically ordered insulator. By applying a magnetic field the structure can be converted to a metal. In

addition, capacitance measurements can be used to measure the “addition” energy of adding an extra electron to the quantum dot array.

We would like to discuss decoherence or the effect of noise on the proposed device. In a universal quantum computer not only the devices representing q-bits are required to be free from noise and decoherence but, in addition, one needs to be able to manipulate them without destruction of coherence. The latter issue obviously does not arise in our case of the dedicated quantum simulator. The proposed device, however, and the original system are both affected by decoherence effects. The CuO planes are hardly in a vacuum, phonons being not far in energy from the energy separation between the electronic states. What is being proposed is a device for simulating a particular model Hamiltonian, which may have many features in common with 2D lattice models. So the question is to what extent one can neglect noise and get reliable answers concerning mostly thermodynamic information that is discussed in this paper. One expects that there will be regions of the thermodynamic phase diagram of the proposed device which will be strongly affected by impurities, imperfections and other sources of noise and regions which will not be strongly affected. The full calculation of the effects of such noise on the phase diagram of these models is left as an open problem; to answer it one needs to carry out a more complete calculation of the electronic properties of the proposed device and their sensitivity to the various sources of noise.

In summary, we have shown that the 2D quantum-dot array produced by the structure shown in Fig. 1 using the calculated hole sizes and proposed gate voltages and doping values, maps onto the strong coupling limit of the Hamiltonian given by Eqs. (13) and (14). This Hamiltonian is that used to describe the physics of the Cu-O plane of the copper-oxide superconductors. This quantum dot array can be considered as an “analog” quantum computer (as opposed to digital) or a dedicated quantum simulator of the dynamics of the Cu-O planes in these materials. In particular we have shown that the striped state formed in the above materials, which seems to be hard to study experimentally,^{13,21} can be studied and analyzed using an analog system of such a 2D quantum dot array which is predicted to form a stripe state with a wavelength of mesoscopic size. This allows conventional imaging methods such as EFM to be used.

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REFERENCES

1. D. P. DiVincenzo, *Nature* **393**, 113 (1998). G. Burkard, D. Loss, and D. P. DiVincenzo, *Phys. Rev. B* **59**, 2070 (1999).
2. D. Loss and D. P. DiVincenzo, *Phys. Rev. A* **57**, 120 (1998).
3. R. P. Feynman, *Int. J. Theor. Phys.* **21**, 467 (1982).
4. S. Lloyd, *Science* **273**, 1073 (1996).
5. D. S. Abrams and S. Lloyd, *Phys. Rev. Lett.* **79**, 2586 (1997).
6. D. A. Lidar and O. Biham, *Phys. Rev. E* **56**, 3661 (1997).
7. J. D. Jackson, *Classical Electrodynamics*, J. Wiley and Sons (1975).
8. S. Tarucha *et al.*, *Phys. Rev. Lett.* **77**, 3613 (1996).
9. P. Hawrylak, *Phys. Rev. Lett.* **71**, 3347 (1993).
10. J. J. Palacios, *et al.*, *Phys. Rev. B* **50**, 5760 (1994).
11. M. Stopa, *Phys. Rev. B* **54**, 13767 (1996).
12. In-Ho Lee *et al.*, *Phys. Rev. B* **57**, 9035 (1998).
13. J. M. Tranquada, B. J. Sternlieb, J. D. Axe, Y. Nakamura, and S. Uchida, *Nature* **375**, 561 (1995).
14. C. S. Hellberg and E. Manousakis, *Phys. Rev. Lett.* **83**, 132 (1999).
15. V. J. Emery, S. A. Kivelson, and H. Q. Lin, *Phys. Rev. Lett.* **64**, 475 (1990).
16. C. S. Hellberg and E. Manousakis, *Phys. Rev. Lett.* **78**, 4609 (1997).
17. F. C. Zhang and T. M. Rice, *Phys. Rev. B* **37**, 3759 (1988).
18. S. R. White and D. J. Scalapino, *Phys. Rev. B* **60**, 753 (1999); *Phys. Rev. Lett.* **80**, 1272 (1998); **81**, 3227 (1998).
19. S. R. White and D. J. Scalapino, *Phys. Rev. Lett.* **84**, 3021 (2000). C. S. Hellberg and E. Manousakis, *Phys. Rev. Lett.* **84**, 3022 (2000).
20. H. O. Jacobs, H. F. Knapp, and A. Stemmer, *Rev. Sci. Instrum.* **70**, 1756 (1999).
21. H. A. Mook, Pengcheng Dal, F. Dogan, and R. D. Hunt, *Nature* **404**, 729 (2000).
22. F. R. Waugh *et al.*, *Phys. Rev. Lett.* **75**, 705 (1995).
23. L. P. Kouwenhoven *et al.*, *Phys. Rev. Lett.* **65**, 361 (1990).
24. C. A. Stafford and S. Das Sarma, *Phys. Rev. Lett.* **72**, 3590 (1994).
25. R. J. Haug, J. M. Hong, and K. Y. Lee, *Surf. Sci.* **263**, 415 (1992).